Evolution of the Vorticity Contour in Vortex Energy Sources

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Abstract. This article utilizes the physical modeling of a differentially rotating shallow fluid to understand the role of restructuring the vorticity contour in realizing the effects of the evolution of intense atmospheric vortex formations. Also paper analyzes the results of laboratory experiments with a rotating fluid model with the conditions for the formation of the polygonal structure of the vorticity contour on the central part and transitions between the modes of the vortex structures at the periphery. Taking into account the diagram of eddy currents stability established for the model where various cases of instability development are visually indicated with changes in the barotropic velocity shift and angular velocity of the entire system, the selection rules for transition times between central polygonal structures and peripheral motion modes with different numbers of vortices are defined. Assuming the analogy and general principles of formation found during radio-location Doppler sounding of polygonal structures of the eye of hurricanes and accompanying peripheral vortices with a corresponding index of symmetry, illustrative calculations of the characteristic times of structural transformations of the configuration of the hurricane eye contour are presented until just before the destruction. Obtained results are important for the organization of the research and monitoring of tropical cyclones including space-based methods.

Keywords: Tropical cyclone · Contour of vorticity · Physical modeling · Polygonal eye · Tropical cyclone genesis · Hurricane monitoring indicator

1 Introduction

Intensive atmospheric vortices in the process of their evolution experience a structural reorganization of internal motions. The most distinct structural changes in the momentum and energy transfer processes are reflected in the transformation of motions in the vicinity of the region of maximum winds [1].

The transformations in the evolution of polygonal configurations of the central part of the trophic cyclones of the so-called wall of the eye, discovered in Doppler radar studies, and the corresponding set of mesoscale spiral vortex structures within the rain

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zone stimulated a spectrum of numerous studies of the mechanisms of occurrence, evolution and destruction of hurricanes [2–6]. Attempts have been made to propose a method for diagnosing cyclogenesis at an early stage of nucleation using the merger index of mesoscale mid-level vortices [7, 8].

The use of the fusion index in mesoscale vortices as a precursor of the formation of hurricanes proved to be a rather suitable means of detecting cyclogenesis at an early stage. The later stages of the evolution of hurricanes up to this day remain a scientific challenge for forecasting and serve as an occasion to consider the processes of reformatting the eye and transitions between different modes of vortex structures behind the hurricanes' eye as a likely general process of the development of intense atmospheric vortices.

This paper proposes an idea to use the method of physical modeling to determine the predictor criteria for the evolution of the vorticity contour of intensive energy sources, such as tropical cyclones. This method despite the absence and impossibility of complete copying of the model from the original allows with the simulation of the main elements of the processes of momentum and energy transfer when simulating to achieve fairly accurate results for the forecast of the development of hurricanes. In this regard, as applied to the search for indicators of the intensification of tropical cyclones, for generalizing consideration, the previously obtained results of laboratory experiments with the model of a differentially rotating shallow liquid were used [9, 10].

In experiments with differentially rotating shallow water, polygonal configurations were observed in the central inner region of the flow [11]. This area corresponded to the processes in the zone of the hurricane's eye. With the change in the magnitude of the horizontal shear rate, the borders of the contour with constant vorticity changed. In the beginning, it was a circle, then an ellipse, triangle, and so on up to a hexagon. Along with the steady state of the polygonal contour, there was a picture of a periodic flip between motion modes with different values of the symmetry index or the mode number of peripheral vortices. Perturbations of the contour, along with the boundary of which the vorticity is constant, are interpreted as the result of the interference of waves developing in a velocity shift [12]. Similar situations are recorded in typhoons. For example, in the Winnie typhoon, radar sensing detected bounces between the four and six-angle "eye" configurations [13]. In model vortices and typhoons, as analysis shows, we are dealing with the same phenomenon, namely, with the inter-transformations between the modes of inertial oscillations [10, 14].

2 Analysis of Wave Contour Restructuring Processes

The results of observations of the temporal dynamics in vortex motions in a physical model of differentially rotating shallow water provide the basis for evaluating the natural transformations of the eye of an intense atmospheric vortex to use characteristic times that are spent on restructuring the vorticity contour. We will designate them as a symbol $\Delta\tau_{ij}$. The first index characterizes the oscillation frequency along the contour. The index value $i = 1$ will denote the state of the contour with low-frequency, long-wave oscillations, which modulated the movement along the contour, and the index

$i = 0$ - high-frequency, short-wave. The second index in the expression for the time of restructuring the contour of the "eye" means the index of symmetry of the polygonal configuration of the contour. It can vary from $j = 0$ (round "eye") to $j = 6$ (hexagonal eye).

Analysis of field observations and laboratory experiments leads to the following conclusion. The times $\Delta\tau_{ij}$ that are spent by differentially rotating shallow water to rebuild the shape of the wave contour are varying linearly with time. There are two types of eye states. They are conventionally designated by us in the form of a system of upper levels

$$\Delta\tau_{1j} = a_{0j} - \alpha t \tag{1}$$

and lower level systems

$$\Delta\tau_{0j} = a_{0j} - \beta t. \tag{2}$$

Here α, β - respectively, the parameters characterizing the rate of adjustment of the contour at the upper and lower levels, a_{0j}, a_{1j} - are the characteristic lifetimes of states with a different configuration of the wave contour.

The flip times at each level corresponding to the energy states of the rotating flow for movements of different scale. The upper row characterizes the amplitude of low-frequency averaged motions of the main flow. The bottom row corresponds to motion states with high-frequency components. In the process of evolution, a systematic transition of the energy of medium (low-frequency) movement into a pulsation (high-frequency) occurs. Then the energy goes partially into heat, and sometimes partially returns from pulsations to the average component. The transition from the upper to the lower levels determines the energy losses of the main flow and the selection of energy from this mode. In this case, a low-level mode is generated, during the development of which a part of the energy returns to the main current.

Consider the condition of transition from one mode to another in the mode of sequential oscillations on the example of typhoon Winnie. The Winnie typhoon in the system of the lower levels, two lines are distinguished $j = 4, j = 5$. For this lines

$$\Delta\tau_{04} = a_{04} + \beta t,\ \ \Delta\tau_{05} = a_{05} + \beta t \tag{3}$$

here $[\Delta\tau] = \text{min}$, $[a] = \text{min}$, $[t] = \text{min}$. Parameter values of a, β the can are found by analyzing the data in the article [13] by the method of least squares. In result: $a_{04} = 9,7$ min, $a_{05} = 9,4$ min, $\beta = 0,12$. Top levels for typhoon include one line with mode $j = 6$

$$\Delta\tau_{16} = a_{16} - \alpha t \tag{4}$$

And $a_{16} = 116\,\text{min}$, $\alpha = 0,2$.

Similar calculations were carried out with a laboratory model. It also confirms the linear law of type (1), (2). You can also talk about the system of upper and lower levels. The main difference between a typhoon and a model is in time scale values. The transition from full-scale to model values, as the comparison showed, should be carried out with the help of a large-scale time coefficient $\xi = 142$ min/s. This factor is obtained from the condition of the correspondence of the times determined by the intersection point of the corresponding lines of Eqs. (1), (2) for the model and the typhoon. Processing the experimental data for the model allowed us to formulate a rule of thumb by which the relative density of the level lines can be found

$$\frac{a_{1j}}{a_{1j+1}} = \frac{j+1}{j-1}. \tag{5}$$

In model experiments, for verification, calculations were made of the characteristic flip times between different states of a polygonal contour. In terms of tropical cyclones, equations of the type (1), (2) taking into account the coefficient ξ, are written as:

$$\Delta\tau_{12} = 77,2 - 0,098t$$

$$\Delta\tau_{13} = 74,3 - 0,098t$$

$$\Delta\tau_{04} = 20,4 + 0,056t.$$

Here all values are expressed in minutes. Table 1 presents a comparison of the calculated values of the flipping times from one contour configuration to another with certain values according to the experimental filming of the model's polygonal "eye" transformations. As can be seen, the calculated values are in good agreement with the experimentally determined values.

Table 1. The characteristic times of the vorticity contour reconstruction in the model system.

№	Mode number	$\Delta\tau$, s	t, s	$\Delta\tau$, min	t, min	$\Delta\tau$, min	t, min
		Experiment				Calculation	
1	3	0,37	0,38	53	53	53	53
	4	0,17	0,55	24	77	25	78
2	3	0,43	0,98	61	138	61	139
	4	0,25	1,23	36	174	30	170
3	3	0,38	1,60	53	227	53	223
	4	0,21	1,81	30	257	30	253
4	3	0,29	2,10	41	298	45	298
	4	0,29	2,39	41	339	40	338
5	3	0,26	2,65	37	376	38	376
	4	0,30	2,95	43	417	43	418
6	3	0,21	3,16	30	449	30	449
	2	0,32	3,48	45	494	47	495

Now let us consider the criteria that can be used to assess the necessary and sufficient conditions for the transfer between the polygonal "eye" configurations. As a result of the analysis of the experimental material obtained in the study of the model system, the following rules were established.

1. The transition from the upper levels to the lower ones is allowed provided the following:

$$\Delta\tau_{1j} > \Delta\tau_{0j} + \frac{\Delta\tau_{0j}}{1-\beta} \quad (6)$$

2. The transition from the lower levels to the upper levels is allowed provided the following:

$$\Delta\tau_{0j} < \Delta\tau_{1j} + \frac{\Delta\tau_{1j}}{1+\alpha} \quad (7)$$

The question of which line will be transferred from the upper levels to one of several lower levels, for which condition (6) is realized, must be solved separately by comparing their energy possibilities. If we compare the two lines j and $j+1$ the lower level, for which condition (6) is satisfied, then for this we should consider the inequality of the form:

$$\Delta\tau_{0j} \geq \frac{\Delta\tau_{0j+1}}{1-\beta} \quad (8)$$

If it is executed, then the transition to the line j rather than to the line $j+1$ is energetically more favourable. Similarly, when transferring from the lower levels to a series of upper lines is analyzed to determine which line j or the line $j-1$ transition will take, it is necessary to check the fulfilment of the next relation

$$\Delta\tau_{1j} \leq \frac{\Delta\tau_{1j-1}}{1+\alpha} \quad (9)$$

When the condition (9) is met, the transition occurs to the line $j-1$. In other cases, i.e. at

$$\Delta\tau_{1j} > \frac{\Delta\tau_{1j-1}}{1+\alpha}$$

the transition from lower levels to j the mode of the upper level. Conditions (8), (9) allow us to estimate the points in time and the critical values of the maximum permissible restructuring times of the typhoon "eye". We denote the limiting adjustment times for the mode j by the lower and upper levels, respectively: $\Delta\tau^*_{0j}$ and $\Delta\tau^*_{1j}$.

These times should be determined, as follows from the results of model experiments, the density of the nearest levels and their slope

$$\Delta\tau^*_{0j} = \frac{a_{0j} + a_{0j+1}}{\beta}; \ \Delta\tau^*_{1j} = \frac{a_{1j-1} + a_{1j}}{\alpha} \tag{10}$$

For Typhoon Winnie, the maximum allowable times for restructuring the polygonal eye of different degrees of symmetry along the lines of the lower level are presented in Table 2.

Table 2. Parameters for restructuring the contour of the eye for Winnie typhoon.

j	3	4	5	6	7
a_{0j}, min	16,4	9,7	6,0	3,8	2,6
$\Delta\tau^*_{0j}$, min	55,8	30,8	18,3	10,6	7,2

Figure 1 shows the data of actually observed values of periods Δτ of rearrangement of the vorticity contour of polygonal eye configurations in the Winnie typhoon for different points of time t of his life in comparison with the calculated lines of the corresponding levels of energy states of movements calculated in accordance with the proposed selection rules.

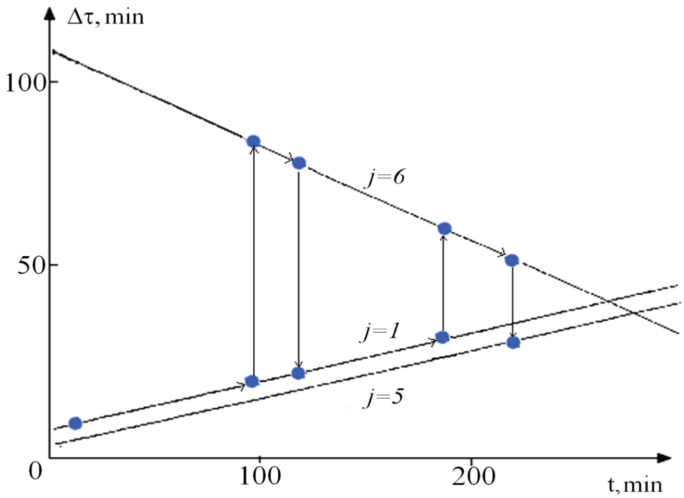

Fig. 1. Schematic representation of the dependence of the period of adjustment of the polygonal eye contour in Winnie's typhoon on the time of his life. Solid lines are calculated by formulas (1), (2). Circles are observational data [13]. The arrows indicate the direction of the structural adjustment of the vortex motions in the typhoon.

The results are in accordance with the observational data given in [2]. A general conclusion that should be made on the basis of the research conducted is as follows. The estimated algorithm for calculating the time of adjustment of the “eyes” of typhoons can be used for analytical verification in prognostic estimates of the evolution of various stages of development of hurricanes.

3 Conclusion

The submissions provide the basis for the following conclusion. As applied to the description of the processes of structural rearrangement of internal motions in tropical cyclones, we used the results of physical modeling of vortex formations in differentially rotating shallow water. Parallel changes in the polygonality of the vorticity contours in the model with the phenomenon of temporary transformation of the typhoon eye were made. Also, polygonal modifications of the eye are compared. In models and intense atmospheric vortices, the polygonality of the eye can be associated with the inter-conversion between the modes of gravitational-inertial oscillations. The characteristic lifetimes of states with different structures of the wave contour in the centre of the vortex and typhoon are found. Paper formulated the rules that determine the conditions for the transfer between the polygonal forms of the eye of the vortex. The proposed algorithm for calculating the restructuring times of the “eye” of intense atmospheric vortices can be used for analytical verification in prognostic estimates of the evolution of various stages of hurricane development.

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